

Winter 2013 MAT 334 (LEC5010) Exam 2

You have 50 minutes. Answer 4 of the following 5 questions. If you answer all 5, your score will be determined by the best 4 solutions you provide.

Problem 1. Consider the following power series about $z_0 = 0$:

$$\sum_{n=0}^{\infty} (2 \cdot 3^n \cdot z^n).$$

What is its radius of convergence?

Problem 2. Find the entire function $f = u + iv$ such that:

- $f(0) = 2$
- The imaginary part of f satisfies $v(x, y) = x$.

State your final answer for f in terms of z , not x and y .

Problem 3. Let γ denote the positively oriented boundary of a triangle with vertices at $-1 - i$, $1 - i$, and i . Evaluate

$$\int_{\gamma} \frac{dz}{z^2 + 2iz}.$$

Problem 4. Suppose $f : \mathbb{C} \rightarrow \mathbb{C}$ is entire, and assume that $|f(z)| > 1$ for all $z \in \mathbb{C}$. What can you say about f ?

Problem 5. Recall that the **index** of a curve γ with respect to $z_0 \in \mathbb{C}$ is

$$n(\gamma; z_0) = \frac{1}{2\pi i} \int_{\gamma} \frac{dz}{z - z_0}.$$

Evaluate $n(\gamma; 0)$, where γ is the following piecewise smooth, closed curve:

$$\gamma : [0, 6] \rightarrow \mathbb{C}, \quad \gamma(t) = \begin{cases} (1+t)e^{2\pi it} & 0 \leq t \leq 5, \\ 6 - 5(t-5) & 5 \leq t \leq 6. \end{cases}$$